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Imagination in Scientific Reasoning: Theoretical Innovation and Scientific Explanation on the Example of Atomic Physics

Imaginación en el razonamiento científico: innovación teórica y explicación científica en el ejemplo de la física atómica

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Abstract

This article aims to show the great variety of ways in which new ideas enter science; ideas that must pass both the critical examination of the scientific community and the filter of empirical contrast. The community itself is aware of the role that conjectures, intuitions, and imagination play in the advancement of science. A fundamental role in the context of discovery is played by abduction, a form of reasoning that serves the purposes of theoretical innovation and scientific explanation, which I illustrate by focusing on the anomalous Zeeman effect, which is very appropriate to the case. But I also point out that in preductive reasoning, a genuinely deductive form of inter-theoretical reasoning, imagination, and luck should not be excluded when choosing the theoretical elements whose step-by-step combination leads to a novel theoretical result.

Keywords: essays, conjectures, hypothetical reasoning, creative imagination in science, theoretical innovation, scientific discovery, explanation, history of ancient atomic physics, the anomalous Zeeman effect, abductive reasoning, theoretical preduction.

Resumen

El objetivo de este artículo es mostrar la gran variedad de maneras en que las nuevas ideas acceden a la ciencia; ideas que han de pasar tanto el examen crítico del colectivo científico como el filtro de la contrastación empírica. La propia comunidad es consciente del papel que las conjeturas, las intuiciones y la imaginación juegan en el avance de la ciencia. Un rol fundamental en el contexto del descubrimiento lo juega la abducción, una forma de razonamiento que sirve a los efectos de innovación teórica y explicación científica, que ilustro centrándome en el efecto Zeeman anómalo, que es bastante apropiado para el caso. Pero también señalo que en el caso del razonamiento preductivo, una forma genuinamente deductiva de razonamiento interteórico, no se debe excluir a la imaginación y la suerte a la hora de elegir los elementos teóricos cuya combinación, paso a paso, conduce a un resultado teórico novedoso.

Palabras clave: ensayos, conjeturas, razonamiento hipotético, imaginación creativa en ciencia, innovación teórica, descubrimiento científico, explicación, historia de la física atómica antigua, el efecto Zeeman anómalo, el razonamiento abductivo, la preducción teórica.

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In memory of Virginia.

Introduction

In his review of Sin-itiro Tomonaga's book, *The Story of Spin* Herman Feshbach (1998, p. 840) recognizes that experimental physicists often pay little attention to the history of their field and that in any case they soon forget how and by what means we have reached the present moment. I am guided in this work by a philosophical interest in the topic of scientific discovery. History has a considerable presence in it, at the service of a question specific to the methodology of science, such as the weight of imagination in scientific innovation and explanation.

But what do we talk about when we think about science? Although I am fully aware that a general characterization of what I understand by science exceeds the main objective of this work, I believe that it does not hurt to make its presentation, knowing, of course, that I do not intend to achieve exhaustiveness in my characterization of science.

I distinguish between science as a human *practice or activity* and science as a *product or result* of this activity. Understood as practice, I conceive science as an activity developed individually and collectively with full methodological rigor and intellectual honesty; rooted in and conditioned by a preceding history; focused on solving specific problems and capable of providing innovation and explanation; guided by the search for truths about the world, or, more modestly, for success in our intellectual interaction with Nature, as well as a better insertion of our species in its environment and scrupulous care of it, in order to preserve it for future generations.

The result of scientific practice is science as a *product*, namely, a growing set of conjectures, hypotheses, theories, and models about the most varied aspects of the world, provisionally accepted with greater or lesser conviction, enthusiasm, and involvement, susceptible to severe empirical testing, and which, at best, seem to reveal to us aspects of the reality to which we belong.

Specifically, this article links with the problems in theoretical physics of the early 20th century, with which Kuhn concludes his detailed 1978 book on black body theory and quantum discontinuity. But I do not intend to complete or complement this splendid text, with which at the beginning of this work there is an inevitable overlap. Simply what interests me here is what Kuhn did not develop in his book. My aim is to present and highlight, through a series of episodes from the history of primitive atomic physics, extended in time during the first quarter of the 20th century, the role that hypothetical reasoning and imagination play in science, sometimes in the form of daring, almost forced conjectures, either in the form of a trial or a lucky bet, in short, in the form of the search for the best tentative explanation capable of accounting for sometimes disconcerting observations and experimental results.

In support of what I say, I want to first bring up very briefly the opinion of Peter Medawar (1974). Relying on authors such as Charles S. Peirce, William Stanley Jevons, William Whewell, and Claude Bernard, Medawar (1974, p. 284) emphasizes, among other things, that "The generative or elementary act in discovery is 'having an idea' or proposing a hypothesis." With the condition that "Hypotheses must be tested, that is criticized." For Medawar (1974, p. 289), "the activity that is characteristically scientific begins with an explanatory conjecture which at once becomes the subject of an energetic critical analysis." This opinion is also held by Henri Poincaré (1963, pp. 110–111), who in the section "The role of the hypothesis" asserts that every hypothesis "must always be subjected to verification as quickly and frequently as possible. It goes without saying that if it does not withstand that test it must be abandoned without reservation." Also, as early as 1870, John Tyndall (1820–1893)—known in the history of physics for the effect named after him—highlights the role of imagination in science. It is true that he does not make a systematic treatment of the subject, but rather veiled allusions, but John Arthur Thomson (1861–1933) in his section "The scientific use of the imagination" (1911, p. 64–65) claims Tyndall's figure. By the way, Thomson (1911, p. 59) also combines scientific imagination and hypothesis. And even Karl Popper (1980) accepts that every discovery contains a "creative intuition."

In the way in which these hypotheses are reached, *abductive reasoning* plays a crucial role, which, in addition to integrating the imaginative resource, contributes explanatory will to scientific innovation. The recognition of the weight of abductive reasoning has awakened the use of what we could call conjectures and trials in the methodology of science. The origins of atomic physics offer evidence that Medawar could have provided in favour of his point of view on the relevance of the role of imagination and hypothetical trial in scientific creativity, which for Wolfgang Pauli (1946, p. 215) was clear: "The essential advance of physics rests on the *creative imagination* [emphasis added] of the experimental as well as the theoretical investigator, and [...] cannot be forced by planning on a grand scale."

Sections 2 and 3 show the path that early quantum physics took until the consolidation of the Planck action quantum hypothesis and the discontinuous distribution of energy. They offer a paradigmatic case study on how the use of intuition, hypothetical reasoning, and imagination, accompanied by essential empirical success, facilitate the incorporation of an innovative theoretical resource in science. Section 4 includes the incorporation of Planck's theory in the construction of Bohr's atomic model, which consolidates the Planckian hypothesis without detracting from the resort to hypothetical reasoning and personal commitment on the part of Bohr. Section 5 focuses on the historic-philosophical presentation of an experimental result, the anomalous Zeeman effect, whose explanation would end up surpassing Bohr's atomic model and its replacement by a broader theory, now free of all ties with classical physics. From the point of view of the methodology of science, the anomalous Zeeman effect fits like a glove for the implementation of abductive reasoning. Finally, section 6 presents a very specific case of contemporary physics, the theoretical anticipation of the possible discontinuity of space, for whose deduction the recourse to luck/imagination also counts.

Conjectures, Trials, and Hypotheses in the Postulation of Planck's Action Quantum

Theoretical physics at the end of the 19th century required renewal, due to the accumulation of problems it faced. The main one in those years was to find an explanation for *black body radiation*. Gustav Robert Kirchhoff (1824–1887) had given the name *black body* to those objects that absorb all the radiation that falls on them, but that, depending on their temperature, can also emit radiation; The striking property that black bodies present is that, at the same temperature, and regardless of their nature, they all radiate in the same way.

Joseph Stefan (1835–1893) had established empirically in 1879 that the total radiation emitted by a black body was proportional to the fourth power of its temperature. Only five years later (1884), Ludwig Boltzmann (1844–1906), applying the laws of thermodynamics to a gas of electromagnetic radiation, achieved the derivation of *Stefan's Law,* and with this, he provided a theoretical explanation for this phenomenon. Ten years later, Wilhelm Wien (1864–1928) obtained, also by thermodynamic procedures, the first law on the energy density of a black body, which made it possible to deduce a law according to which as the temperature of the black body increases its radiation shifts towards shorter wavelengths. For this reason, it became known as *Wien's Displacement Law*.

Now, as Max Planck (1949, p. 21) acknowledged, Otto R. Lummer, Ernest G. Pringsheim, and Friedrich Paschen had shown that the application of Wien's Law to longer wavelengths raised serious doubts. There was indeed an attempt to overcome this situation, the *Rayleigh-Jeans Law*, but the problem with this law was that at high frequencies it failed miserably, a situation that came to be called the *ultraviolet catastrophe*. Planck (1900, p. 1) himself in his presentation in Berlin before the German Physical Society on October 19, 1900, states that Lummer and Pringsheim, Kurlbaum and Rubens, and Paschen, confirmed: "that Wien's energy distribution law [...] at most has the character of a limiting case, the simple form of which was due only to a restriction to short wavelengths and low temperatures." Or, as he also states: "Whilst for small values of the energy and for short waves, Wien's law was satisfactorily confirmed, noteworthy deviations for larger wavelengths were found, first by O. Lummer and E. Pringsheim, and finally by H. Rubens and F. Kurlbaum" (Planck, 1920, para. 9). So, since the most that thermodynamics and electromagnetism had managed to offer was two laws that partially saved the phenomenon, one for short waves and the other for long waves, the empirically verified black body radiation still lacked a satisfactory explanation.

And this is where the long push and pull in using imagination begins. In a *first approximation*, intuition led Planck to assume that the intensity of the radiation was related to the dependence of the entropy on the energy—"I assumed the most profound relationship in the dependence of the entropy S on the energy U" (Planck, 1949, p. 21)—[1]which led him (Planck, 1900, p. 2) to express Wien's law in the form

$$\frac{d^2 S}{dU^2} = \frac{const}{U}$$

However, Planck (1900, p. 2):

> *I consider the possibility* [emphasis added], even if it would not be easily understandable and *in any case would be difficult to prove* [emphasis added], that the expression on the left-hand side would not have the general meaning which I attributed to it earlier. [...] Following this suggestion, I have finally started *to construct completely arbitrary expressions* [emphasis added] for the entropy which although they are more complicated than Wien's expression, still seem to satisfy just as completely all requirements of the thermodynamic and electromagnetic theory.

In particular, continues Planck (1900, p. 2):

> *I was especially attracted by one of the expressions thus constructed* [emphasis added] which is nearly as simple as Wien's expression and which deserves to be investigated [...] We get this expression by putting
>
> $$\frac{d^2 S}{dU^2} = \frac{\alpha}{U(\beta + U)}$$
>
> It is by far the simplest of all expressions which lead to S as a logarithmic function of U [...] and which moreover reduces to Wien's expression for small values of U.

Well, combining the definition of temperature with Wien's radiation law, Planck (1900, p. 3) arrived at his radiation formula

1. "*Ich in der Abhängigkeit der Entropie S von der Energie U den tieferen Zusammenhang vermutete.*"

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> Which, as far as I can see at the moment, *fits the observational data, published up to now* [emphasis added], as satisfactory as the best equations put forward for the spectrum […], [which] I consider to be the simplest possible, apart from Wien's expression, from the point of view of the electromagnetic theory of radiation.

Now, as Planck (1920, para. 11) himself confesses in the reading for the awarding of the Nobel Prize: "Even if the radiation formula should prove itself to be absolutely accurate, it would still only have, within the significance of a *happily chosen* [emphasis added] interpolation formula, a strictly limited value." As Ilse Rosenthal-Schneider (1949, p. 68) points out: "A *semi-empirical* approach did not satisfy Planck any more than juggling with pure mathematical formulas."

The *second step* is described by Planck (1920, para. 11) himself in the following way:

> For this reason, I busied myself […], from the day of its establishment, with the task of elucidating a true physical character for the formula, and this problem led me automatically to a consideration of the connection between entropy and probability, that is, Boltzmann's trend of ideas; until after some weeks of the most strenuous work of my life, light came into the darkness, and a new undreamed-of perspective opened up before me.

A clear case of the success of the use of imagination, conjectures, and trials in the search for the best possible explanation of observational data.

On June 1, 1920, the Royal Academy of Sciences decided to award the 1918 Nobel Prize in Physics to Max Planck for his work in the establishment and development of the theory of quanta:

> Planck's radiation theory is, in truth, the most significant lodestar for modern physical research, and it seems that it will be a long time before the treasures will be exhausted which have been unearthed as a result of Planck's genius. (Ekstrand, 1920, para. 3)

In Rivadulla (2002, pp. 54–55), I point out that Planck's radiation law was empirically progressive: it explains different and partly unconnected hypotheses, since the Wien displacement law, the Stefan-Boltzmann law, and the Rayleigh-Jeans law are mathematically derived from it; the last one as a limiting case of Planck's law. Furthermore, the quantum hypothesis makes it possible to explain phenomena that until then had no justification, such as the photoelectric effect.

In relation to the quantum of action, Planck (1920, para. 16) manifests himself in the following way:

> Because it represents the product of energy and time (according to the first calculation it was 6.55×10^{-27} erg sec), I described it as the elementary quantum of action […] it proved elusive and resistant to all efforts to fit it into the framework of classical theory.

The reason for his naming of the constant *h* as the elemental quantum of action or action element (Wirkungselement) was that "it has dimensions of the product of energy and time"[2] (Planck, 1949, p. 26).

For Planck (1920, para. 16) the choice was clear:

> Either the quantum of action was a fictional quantity, then the whole deduction of the radiation law was in the main illusory and represented nothing more than an empty non-significant play on formulae, or the derivation of the radiation law was based on a sound physical conception. In this case, the quantum of action must play a fundamental role in physics, and here was something entirely new, never before heard of, which seemed called upon to basically revise all our physical thinking.

2. "*Sie die Dimension eines Produktes von Energie und Zeit besitzt.*"

Planck's position was that "Experiment has decided for the second alternative" (Planck, 1920, para. 17). A decision based on the "restless forward thrusting work of those research workers who used the quantum of action to help them in their own investigations and experiments" (para. 17).

According to Arnold Sommerfeld (1923, p. 37):

> [Planck] was himself compelled to take a *bold step* leading away from the main road of our usual way theory and to propound his *hypothesis of energy-quanta*. He *postulated that energy of radiation of any frequency v whatsoever can be emitted and absorbed only in whole multiples of an elementary quantum of energy* $\varepsilon = h\nu$.

He adds that "This hypothesis is the foundation of the photo-electric law of Einstein and also of its extension as Bohr's hypothesis concerning emitted and absorbed energy" (Sommerfeld, 1923, p. 193).

Barely after five years, the quantum of action was already used as an available resource for theoretical explanation of phenomena orphans of it. Planck (1920, para. 17) recognizes that

> The first impact in this field was made by A. Einstein who […] pointed out that the introduction of the energy quanta, determined by the quantum of action, appeared suitable for obtaining a simple explanation for a series of noteworthy observations during the action of light

And that "the quantum hypothesis has, nevertheless, its greatest support from the establishment and development of the atom theory by Niels Bohr" (Planck, 1920, para. 22). An extremely important consequence of this: "The first brilliant acquisition was the derivation of Balmer's series formula for hydrogen and helium including the reduction of the universal Rydberg constant to merely known numerical quantities." That is its explanation within the framework of Bohr's atomic model.

In conclusion,

> After all these results […] there is no other decision left *for a critic who does not intend to resist the facts* [emphasis added] than to award to the quantum of action […] full citizenship in the system of universal physical constants. (Planck, 1920, para. 24)

It is true that, at the beginning, as Planck (1949, p. 27) himself acknowledges, the value of the constant h "was completely up in the air."[3] But he adds:

> That is why it was a source of great surprise and joy for me when J. Franck and G. Hertz found a method for its measurement in their experiments on the excitation of a spectral line by means of electron collisions, as could not be wished more directly. (p. 27)

In fact, from the results of measurements in collisions between electrons and mercury atoms, Franck and Hertz (1967) concluded that $h = 6.59 \times 10^{-27}$ erg seg with a possible two percent error.

The acceptance of the elementary quantum of action opened the doors to quantum physics, despite all of Planck's unsuccessful attempts to make it fit into the framework of classical theory. Planck's radiation law and its explanation acquired the rank of revolutionary innovation.

Einstein's Support for Planck's Quantum of Action and the Quantization of Energy

At the beginning of the 20th century Albert Einstein (1967, p. 133) already resorted to the quantum of action to maintain the hypothesis of the discontinuous propagation of light:

3. "*Völlig in der Luft hing.*"

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> It seems to me that the observations on "black radiation," photoluminescence and production or transformation of light, seem better understandable under the *assumption that light energy is distributed in space discontinuously*. According to this assumption, the propagated energy of a ray of light starting from a point is not distributed continuously in increasingly larger spaces, but *itself consists of a finite number of energy quanta* [*Energiequanten*] located in points in space, which move without dividing and can be absorbed and produced as a whole.

The Planckian concept of energy quantum, applied both to blackbody radiation and to other light phenomena, was therefore assumed by Einstein only five years after Planck's proposal.[4]

Planck (1967) had established the relationship

$$\frac{h}{k} = 4{,}866 \text{ x } 10^{-11}$$

between his action quantum h and Boltzmann's constant k, so $h = 6{,}55 \times 10^{-27}$ erg. seg. This is important to take into account because, in the first article in which a claim for the quantum of action is made, which is that of Albert Einstein (1967), he uses the expression β with the value $4{,}866 \times 10^{-11}$; that is

$$\beta = \frac{h}{k}$$

In other words, Einstein unquestionably makes use of Planck's action quantum, which means that h began to increase the number of universal natural constants and with it the idea of the quantization of energy, surprisingly quickly.

Einstein's aim (1967, pp. 143–144) was "to find out whether also the laws of the production and transformation of light are made as if light consisted of such quanta of energy." Einstein's conclusion (1905, pp. 145–146) is:

> According to the conception that incident light consists of quanta with energy $(R/N)\beta\nu$, the production of cathode rays by means of light is understood as follows: Energy quanta enter the surface layer of the body, the energy of which is transformed, at least in part, into kinetic energy of electrons. [...] An electron inside the body provided with kinetic energy will have lost part of its kinetic energy when it has reached the surface.

This is the origin of the explanation of the *photoelectric effe*ct. The use of the concept of energy quantum and Planck's constant h became essential just five years after their incorporation into theoretical physics.

First Approaches in Atomic Physics: Bohr's Predecessors, Bohr Himself, and the Consolidation of Planck's Hypothesis

Although in the years 1905 and 1906, as Kuhn (1980, pp. 266–268) recognizes, the quantum was hardly in the professional consciousness around 1911 and 1912, from 1912 onwards there were very few physicists who, belonging to any of the main centres, continued to ignore or disdain the quantum.

Bohr's Predecessors: Arthur Haas and John W. Nicholson

From 1905 onwards, barely five more years elapsed before the hypothesis of the quantization of energy found the new field in which it took root fruitfully and definitively: the

4. Section 2 of Einstein (1967, pp. 95–96) is titled "On the Determination of Planck's Elementary Quanta."

incipient atomic physics. Armin Hermann (1965, p. 255) expresses himself unequivocally in this regard as follows: "It was Arthur Eric Haas (1894–1941) who, for the first time, applied Planck's quantum of action to the construction of the atom." Niels Bohr (1913, p. 6) mentions that Haas, and Arnold Sommerfeld (1923, pp. 216–217), recognizes referring to the Rydberg constant, that "Before Bohr, A. E. Haas, in particular, had already proved the universal nature of this constant, and had shown how it was very probable that it could be expressed in terms of *h* and electronic data." And Friedrich Hund (1978, p. 159) also states, without further explanation, that "A direct connection of *h* with an atomic magnitude was given by A. E. Haas (1910). He equated 'according to quantum theory' to *hv*[…] the energy in Thomson's then usual atomic model with one electron."[5]

In some unpublished autobiographical notes, echoed by Hermann (1965, p. 260), Haas in effect confesses what he considered the greatest achievement of his life:

> I was the first to apply the quantum hypothesis to the problems of atomic structure and I was the first to find the relationship of the fundamental spectroscopic constants with the quantum of elementary action, the charge of the electron and the mass of the electron.

So, as Hund (1978, p. 160) states: "*h* became a natural unit, which together with *e* and *m*, determined the size of the atoms."[6] I have not been able to find a justification or clarification by Haas himself of this first use of *h* in atomic physics.

Another step forward was taken by the English mathematician and physicist John William Nicholson (1881–1955). In his article "The Constitution of the Solar Corona," Nicholson (1912, p. 677) assumes that the spectrum of the elements can be expressed in terms of Planck's constant and that Planck's theory—one of the objectives of the article is "giving to Planck's theory an atomic foundation"—"states that the energy possessed by a resonator is of the form *nhv*, where *n* is the frequency of the resonator, *n* is an integral number, and *h* is an universal constant of nature." Well, what this means for Nicholson is that "interchanges of energy are not continuous." Nicholson (1912, p. 679) maintains that the ratio of energy to frequency is proportional to the angular momentum of the electrons around the nucleus, which leads him to the conclusion that "If, therefore, the constant *h* of Planck has, as Sommerfeld[7] has suggested, an atomic significance, it may mean that the angular momentum of an atom can only rise or fall by discrete amounts when electrons leave or return." An idea he repeats in the Summary of his article: "an expulsion or retention of an electron by an atom probably involves a discontinuous change in the angular momentum of the atom" (p. 692). What would be reflected in the spectral lines of the atoms.[8] In this regard, Sommerfeld (1923, p. 212) recognizes that "before Bohr, J. W. Nicholson (1912, p. 679) set up the quantum condition for the rotator and used it to interpret certain lines of the sun, as well as of nebulae." This is also the case of Bohr (1913, p. 6) who affirms that

> Nicholson has obtained a relation to Planck's theory showing that the ratios between the wavelength of different sets of lines of the coronal spectrum can be accounted for with great accuracy by assuming that the ratio between the energy of the system and the frequency of rotation of the ring is equal to an entire multiple of Planck's constant.

5. "*Eine direkte Verknüpfung von h mit einer atomaren Grösse gab A. E. Haas (1910). Er setzte beim damals üblichen Thomsonschen Atommodel mit einer Elektron […] die Energie 'nach der Quantentheorie' gleich hv.*"
6. "*h wurde eine natürliche Einheit, die zusammen mit e und m die Grösse der Atome bestimmte.*"
7. In 1911, Arnold Sommerfeld had published an article entitled "Planck's Action Quantum and Its Overall Importance for Molecular Physics," which according to Nelson H. F. Beebe (2017, pp. 122–123) in his *A Complete Bibliography of Publications by, and about, Arnold Sommerfeld*," would have appeared in various media.
8. The importance of the quantization of angular momentum will be emphasized in section 5, "The Impact on Early Atomic Physics of the Anomalous Zeeman Effect: A Three-Step Application of Abductive Reasoning for Its Explanation," in relation to Zeeman's anomalous effect.

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In conclusion, at the beginning of the second decade of the 20th century, Planck's assumption of the quantum of action constituted an available theoretical resource capable of contributing to the best tentative explanation of the observable facts that concerned physicists before Bohr. It is also clear that Nicholson's incorporation of Planck's quantum theory into atomic theory was decisive for Bohr's elaboration in 1913 of his atomic model which incorporates the action quantum.

Hypothetical Reasoning in the Construction of Bohr's Atomic Model

The Danish physicist Niels Bohr (1885–1962) accepted the challenge of modelling a stable atom in 1913. Indeed, in his well-known trilogy "On the Constitution of Atoms and Molecules," Bohr (1913, p. 2) recognizes "the inadequacy of the classical electrodynamics in describing the behaviour of systems of atomic size," so "it seems necessary to introduce [...] a quantity foreign to the classical electrodynamics, i.e., Planck's constant, or as it often is called the elementary quantum of action," so that its application "to Rutherford's atom-model affords a basis for a theory of the constitution of atoms." For Bohr (1913, p. 4) in particular

> The essential point in Planck's theory of radiation is that the energy radiation from an atomic system does not take place in the continuous way assumed in the ordinary electrodynamics, but that it, on the contrary, takes place in distinctly separated emissions, the amount of energy radiated out from an atomic vibrator of frequency v in a single emission being equal to τhv, where τ is an entire number, and h is a universal constant.

The two principal assumptions that Bohr (1913, p. 7) makes use of are 1. That the passage of systems between different stationary states cannot be treated based on ordinary mechanics, and 2. That this passage "is followed by the emission of a *homogeneous* radiation, for which the relation between the frequency and the amount of energy emitted is the one given by Planck's theory." This is an assumption, that Bohr (1913, p. 7) affirms, "*appears to be necessary in order to account for experimental facts* [emphasis added]."

Well, according to Bohr (1913, p. 5), it was precisely Einstein who was the first to recognize "The general importance of Planck's theory for the discussion of the behaviour of atomic systems." By the way, on page 17, Bohr refers again to the expression, which he recognizes as deduced by Einstein, for the kinetic energy of an electron ejected from an atom by the photoelectric effect.

But there is still a third assumption, the quantization of the angular momentum of the electron, for whose presentation Bohr (1913) waits for page 15, where he states that

> The angular momentum of the electron round the nucleus in a stationary state of the system is equal to an entire multiple of a universal value, independent of the charge of the nucleus. The possible importance of the angular momentum in the discussion of atomic systems in relation to Planck's theory is emphasized by Nicholson.

This has already been shown here, as the basic line of Nicholson's theory was outlined.

In any case, and important concerning the objectives of this article, Bohr (1913, p. 19) once again insists on the fact that

> The preliminary and *hypothetical character* [emphasis added] of the above considerations needs not to emphasized. The intention, however, has been to show that the sketched generalization of *the theory of the stationary states possibly may afford a simple basis* [emphasis added] of representing a number of experimental facts which cannot be explained by help of the ordinary electrodynamics, and that *the assumptions used do not seem to be inconsistent with experiments* [emphasis added] on phenomena for which a satisfactory explanation has been given by the classical dynamics and the wave theory of light.

This way of arguing from the current perspective on the methodology of science is that of genuinely abductive reasoning.

The Consolidation of the Action Quantum in Bohr's Atomic Model

As is known, the consolidation of Bohr's atomic model and therefore the culmination of old quantum physics is the work of Arnold Sommerfeld. Section 3 of Chapter IV is titled by Sommerfeld (1923): "Bohr's Theory of Balmer's Series." Starting from the assumption of a single electron that rotates circularly around the nucleus of the atom, which instantiates the model of a simple rotator, Sommerfeld (1923, pp. 211–212) states that "The orbit of the electron is fixed by two conditions, one prescribed by the classical theory, the other by the quantum theory." Thus, "The classical theory requires—claims Sommerfeld—that the external forces be in equilibrium with the inertial forces." What this means is, as Rivadulla (2003, pp. 178–179) states following typical presentations, that "the Coulomb force between the electron and the nucleus is balanced by the centrifugal force due to the circular motion" of the electron; on the other hand, the angular momentum of the electron is quantized in whole numbers of $\hbar$, being $\hbar = h/2\pi$, also called reduced Planck constant. Both conditions together require "*that the electron moves only in certain 'quantised' circles on the* 1st, 2nd, …, *nth* '*Bohr circle*'; *n* is the 'quantum number' of the orbit." Moreover, known as "Bohr's *frequency condition* for spectral emission," Sommerfeld claims:

> This equation states that if the atom passes over from an initial state of energy W_a to a final state of lesser energy W_e, then the excess of energy is radiated out in the form of a monochromatic wave of light, the frequency v of which is determined by just equ. (14) [$hv = W_a - W_e$]. Each such transition thus causes an emission of well-defined light and is observed as a sharp line.[9]

Sommerfeld's (1923, pp. 215–217) conclusion is that "Bohr's theory is thus confirmed very strikingly."

Twenty years after its proposal by Planck, the quantum of action is already a fully consolidated theoretical concept, so its use does not raise any objections. For example, in 1923 the American physicist Arthur Holly Compton (1892–1962) published an article whose title, which perfectly reflects its content, is "A Quantum Theory of the Scattering of *X*-Rays by Light Elements." In it, Compton (1923, p. 485) already assumes the quantum model from the outset: "From the point of view of the quantum theory, we may suppose that any particular quantum of X-rays is not scattered by all the electrons in the radiator, but spends all of its energy upon some particular electron." Compton's (1923, p. 501) conclusion is that the remarkable agreement between theory and experiment "can leave but little doubt that the scattering of X-rays is a quantum phenomenon." Furthermore, "The experimental support of the theory indicates very convincingly that a radiation quantum carries with it directed momentum as well as energy."

The Impact on Early Atomic Physics of the Anomalous Zeeman Effect: A Three-Step Application of Abductive Reasoning for Its Explanation

Pieter Zeeman (1865–1943) begins his 1897 article with the following words:

> In consequence of my measurements of Kerr's magneto-optical phenomena, the thought occurred to me whether the period of the light emitted by a flame might be altered when the flame was acted upon by magnetic force. It has turned out that such an action really occurs. (p. 347)

9. The subscripts a and e correspond respectively to the German "*anfang*," initial, and "*ende,*" final.

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And as Oliver Lodge (1897, p. 513) says about the "remarkable discovery" of P. Zeeman: "I have set up apparatus suitable-for showing the effect, and have verified its primary feature, viz., that both lines in the ordinary spectrum of sodium are broadened when a magnetic field is concentrated upon the flame emitting the light." And in an extension in December of this same year, Lodge specifies: "when the flame is subjected to a concentrated magnetic field, […] a third bright line, as it were, makes its appearance in the midst of the dark line, giving a triple appearance to each sodium line."

The *Zeeman effect* is known as the doubling of the spectral lines emitted by an atom as a result of the excitation produced in it by the action of an external magnetic field. Max Jammer (1989, p. 121) reports that Lorentz's classic explanation of the Zeeman effect allowed us to conclude that "the fall of 1897 the agreement between experiment and theory was perfect." And Poincaré (1963, p. 159) congratulates himself on "the ease with which the new Zeeman phenomenon has quickly found its place."

The Shock in Atomic Physics due to the Anomalous Zeeman Effect

However, in December of that same year Thomas Preston (1860–1900) published his experiments, which led Jammer (1989, p. 121) to affirm that "Preston's observations of what was subsequently called the '*anomalous Zeeman effect*' [emphasis added]—in contrast to the 'normal effect' which 'conforms' to theory—was soon confirmed by Cornu," so that "From 1898 on, when Lorentz attempted unsuccessfully to interpret Preston's and Cornu's observations by generalizing his theory of the normal effect, until the end of the older quantum theory, *the anomalous Zeeman effect remained an unsolved problem* [emphasis added]." In fact, as Mehra and Rechenberg (1982, p. 446) relate: "instead of always finding the Zeeman triplet, one observed in general what Lorentz called the 'more complex types of the Zeeman effect' or Friedrich Paschen and Ernst Back ultimately named the 'anomalous Zeeman effect.'"

Helge Kragh (2007, p. 153) describes the anomalous Zeeman effect as catastrophic, referring to a letter from Pauli to Sommerfeld in 1923. Actually, claims Kragh (2007, p. 154), there were many "relevant experiments and facts that the Bohr-Sommerfeld theory *was incapable of explaining* [emphasis added] and, in this sense, they were *anomalies* [emphasis added]." And he summarises some lines later:

> In 1924, the accumulation of experimental anomalies, together with a general dissatisfaction with the logical and conceptual structure of existing quantum theory, came to create a crisis situation in the small community of atomic physicists. Several physicists concluded that the Bohr-Sommerfeld theory was hopelessly flawed and should be replaced with some other theory.

Quoted by Mehra and Rechenberg (1982, p. 458), Sommerfeld endorsed in his 1919 book, *Atombau und Spektrallinien*, that "A genuine theory of the Zeeman effect in the case of non-hydrogen-like atoms cannot be given until the reason for the multiplicity of spectral lines has been clarified." (pp. 438–439) The aforementioned Mehra and Rechenberg (1982, pp. 476–477) also conclude: "Landé had not been able to provide a reasonable model for atoms emitting doublet and triplet spectra and having the observed Zeeman effects."

The situation would become even more complicated when the Zaragoza native Miguel Catalán Sañudo (1894–1957) discovered multiplets, a term coined by himself, in 1922 in the manganese spectrum. Some multiplets of other atoms could consist of up to fifteen lines.[10]

10. Mehra and Rechenberg (1982, p. 480) offer a biographical review of Miguel Catalán in footnote 760.

The Use of Abductive Reasoning for the Explanation of the Anomalous Zeeman Effect

The anomalous Zeeman effect, which occurs when a beam of atoms is subjected to the action of a weak magnetic field, appears to the incipient community of atomic physicists as a challenge that urgently demanded an explanation. This phenomenon fits like a glove for its explanation to be accepted as a paradigmatic case of abductive reasoning. Other good examples in the history of the physical and natural sciences are the postulation of double stars and the existence of dark matter in astrocosmology, the continental drift in geophysics, the existence of Homo antecessor and other hominins in paleoanthropology, which I have presented elsewhere (Rivadulla, 2021, 2022).

The logical form of Charles S. Peirce's (1965, CP 5189) abductive argument makes it clear what this form of reasoning has to do with extraordinary cases:

– The surprising fact *C* has been observed.

– But if *A* were true, *C* would be a matter of fact.

Hence: There is reason to suspect that *A* is true.

However, this argument does not pretend to be logically rigorous. It does not conclude the truth but the suspicion of the truth or viability of the imagined hypothesis. Nor does it allow for the possibility of a probabilistic evaluation of the truth of *A*. The hypothesis is stated "only problematically or conjecturally" (Peirce, 1965, CP 5188), so abduction is "[the] step of adopting a hypothesis as being suggested by the facts" (CP 7202). Furthermore, abduction is the argumentative form "which introduces any new idea," and also "the process of forming an explanatory hypothesis" (CP 5171). In other words, abduction fulfils the double task of innovation and explanation in science. The examples of abductive reasoning provided by the history of science show that this way of reasoning is compatible with the revision of previously accepted hypotheses in the light of new data.

From the Anomalous Zeeman Effect to the Electron Spin

Wolfgang Pauli (1946, p. 214), Bohr's collaborator in Copenhagen in 1922, concerned about the *anomalous Zeeman effect*, recalls:

> The *anomalous type of splitting* [emphasis added] was on the one hand especially fruitful because it exhibited beautiful and simple laws, but on the other hand it *was hardly understandable* [emphasis added], since very general assumptions concerning the electron, using classical theory as well as quantum theory, always led to the simple triplet. *A closer investigation of this problem left me with the feeling that it was even more unapproachable* [emphasis added].

And Max Jammer (1989, p. 133), literally quoting Pauli in 1923, notes that he concluded, by critically examining the doublet structure of the alkali spectra, "that the point of view then orthodox—according to which a finite angular momentum of the atomic core was the cause of this doublet structure—must be given up as incorrect." As Mehra and Rechenberg (1982, pp. 671–672) recognize

> The failure of classical mechanics—i.e., of an essential part of the fundamental principles of atomic theory—which was reflected [...] by the inability (of the Bohr-Sommerfeld theory) to explain the phenomena of the anomalous Zeeman effects, made it obvious that the known difficulties of the existing theory could not be resolved by a slight modification of the assumptions or equations that had been used so far.

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The tentative explanation—for a complete theoretical explanation physicists had to wait for Dirac's development of relativistic quantum mechanics—will come from the anticipated assumption, first, of the Exclusion Principle and then of the electron spin hypothesis. The fundamental idea of the Exclusion Principle was published by Pauli in 1925: "the exclusion principle maintains that a box can contain no more than one electron […] Quantum theory maintains that other particles such as photons or light particles show opposite behaviour; that is, as many as possible fill the same box" (as cited in Pauli, 1946, p. 214). The presentation of this principle is offered by Pauli (1925). The trigger lies in the fact, which Pauli (1925, p. 1) mentions, that "especially the doublet structure of the alkali spectra and their anomalous Zeeman effect are caused by a *classically undescribable* [emphasis added] two-valuedness of the quantum theoretical properties of the optically active electron." The addition of the fourth quantum number of magnetic moment, although it was a "tentative point of view," led Pauli to the presentation of the

> More general rule about the occurrence of equivalent electrons in an atom: *There can* never *be two or more equivalent electrons in an atom for which in strong fields the values of all quantum numbers* […] *are the same. If an electron is present in the atom for which these quantum numbers* (*in an external field*) *have definite values, this state is "occupied"* [emphasis added]. (Pauli, 1925, p. 8)

Well, from the point of view of this work, Pauli's (1925, p. 8) following confession, lines later, is quite illustrative: "*We cannot give a further justification for this rule, but it seems to be a very plausible one* [emphasis added]." Which, interpreted in the context of the current philosophy of science, supposes that the Exclusion Principle is the result of an abductive inference. For, as Pauli (1925, p. 9) points out, "the consequences of our rule agree with experiment in the simplest cases." That is, it is an *inference to the best explanation*.

The fact is, as Mehra and Rechenberg (1982, pp. 682–683) recognize, that although "By the end of 1925, the exclusion principle belonged to the accepted laws of atomic physics," "The physicists worked with the exclusion principle in spite of the fact that *Pauli was unable to provide 'a more precise justification' for it*." However, the first difficulties in accepting this principle were overcome, as Pauli (1946, p. 214) elegantly recognizes, thanks to "Uhlenbeck and Goudsmit's idea of electron spin, which made it possible to understand the anomalous Zeeman effect," although as Mehra and Rechenberg (1982, p. 684) point out "The spin, an extra mechanical property of the electron, was not in the spirit of the ideas which the author of the exclusion principle developed."

George Uhlenbeck and Samuel Goudsmit—according to a quote from Uhlenbeck in Max Jammer (1989, p. 144)—recognized the value of Pauli's 1925 article "in which the famous exclusion principle was formulated and in which, for the first time, *four* [emphasis added] quantum numbers were ascribed to the electron," although they state that: "This was done rather formally; no concrete picture was connected with it. *To us this was a mystery* [emphasis added] […] We could understand it only if the electron was assumed to be a small sphere that could rotate."

The honour was thus reserved for Uhlenbeck and Goudsmit, who, cited by Mehra and Rechenberg (1982, p. 694) in a note published in *Naturwissenschaften*, 1915, "proposed to associate Pauli's fourth quantum number with 'an intrinsic rotation of the electron.'" In this regard C. A. Coulson (1952, pp. 30–31) states that

> In 1925 it was shown by Uhlenbeck and Goudsmit […] that in addition to its orbital motion, an electron must be regarded as spinning about some axis, through its centre. This motion gives it an angular momentum and (because the electron is an electrically charged body) a magnetic moment.

Although Lorentz maintained that the rotational speed of the electron would exceed several times that of light in a vacuum, and Pauli and Heisenberg were equally opposed to electron spin, Bohr was in favour. And Pauli (1946, p. 215) himself would end up recognizing that

> As Bohr was able to show on the basis of wave mechanics that the electron spin cannot be measured by classically describable experiments (as, for instance, deflection of molecular beams in external electromagnetic fields) and must therefore be considered as an essentially quantum mechanical property of the electron.

As early as 1927 Walter Heitler and Fritz London explained the H_2 molecule as the covalent chemical bond in which one electron has spun up and the other has spun down (Shaik *et al*., 2021, p. 5).

Uhlenbeck and Goudsmit certainly took the lead, but as already mentioned and Feshbach (1998, p. 839) states, "the electron spin and magnetic moment are results of the *relativistic* quantum mechanics proposed by Dirac (1928)," since not even Erwin Schrödinger's quantum mechanics predicts them. All theoretical advances by Pauli and Uhlenbeck-Goudsmit are therefore the product of hypothetical inferences to the best explanation, that is, abductive reasoning, very useful at the time, and which would end up receiving a full-fledged explanation with subsequent development of Dirac's relativistic quantum mechanics. As Pauli (1996, p. 220) himself recognizes, referring to his exclusion principle, "To seek a *theoretical explanation* [emphasis added] of this law we must move to the field of relativistic wave mechanics since [...], it cannot be explained by non-relativistic wave mechanics." But this is the subject of an independent study that takes this case as an example of theoretical explanation in physics.

The Abductive Explanation of the Anomalous Zeeman Effect in Three Steps

The implementation of abductive reasoning in this episode of the origins of atomic physics occurs in three successive steps, of which the first two summarize what was already exposed in the previous section.

The *first* step is the following:

Surprising fact: Splitting of the spectral lines emitted by atoms subjected to a weak uniform magnetic field (*anomalous Zeeman effect*). For Wolfgang Pauli (1946, p. 214) this was *hardly understandable*, even *unapproachable*, and the result of a *classically undescribable property*.

Pauli's tentative solution: The addition of a fourth quantum number that led him to the postulation of the *Exclusion Principle*: There cannot be two electrons occupying the same state with equal quantum numbers.

Problem: According to Pauli (1925, p. 8) however "We cannot give a further justification for this rule, *but it seems to be a very plausible one*." Nonetheless, "*the consequences of our rule agree with experiment in the simplest cases*" (p. 9).

That is, the postulation of the *Exclusion Principle* clearly constitutes an example of *inference to the best explanation*.

Second step:

New problem: Uhlenbeck and Goudsmit acknowledge the importance of Pauli's article "in which the famous exclusion principle was formulated and in which, for the first time, *four* quantum numbers were ascribed to the electron" (as cited in

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Jammer, 1989, p. 144). But for them "This was done rather formally; no concrete picture was connected with it. *To us this was a mystery*" (p. 144).

Uhlenbeck and Goudsmit's *new hypothesis*: "*We could understand it only if the electron was assumed to be a small sphere that could rotate*" (Jammer, 1989, p. 144). That is to say, the electron possessed a property that until then was unimaginable: spin. A new hypothesis, abductively postulated, completes the previous one.

Third step: It consists of the insertion of the new concept imaginatively introduced, the *spin*, in a manner consistent with the quantum mechanical formalism. Coherence is achieved through the use of an *argument by analogy*. Briefly, it is as follows:

According to Bohr's atomic model for a hydrogenoid atom, the electron rotates in a circular orbit around the nucleus, thereby acquiring an angular momentum L. Now, since it is a charged particle, its movement generates a magnetic dipole, which causes the atom to acquire magnetic moment.

Applying the technique of separation of variables in the time-independent Schrödinger equation, its solutions allow us to deduce the existence of three quantum numbers:

$n = 0, 1, 2, 3, \ldots$ (principal quantum number)

$l = 0, 1, 2, \ldots, n\text{-}1$ (orbital angular momentum)

$m_l = -l, -l+1, \ldots, 0, \ldots +l+1, +l$ (magnetic quantum number)

Angular momentum, like energy, is quantized, that is, it only admits discrete values:

$$L = \sqrt{l(l+1)}\,\hbar$$

But, in addition, its direction is also quantized: Given a Z axis, its values are given by $L_z = m_l\hbar$. So that for each value of L there are $2l+1$ values of m_l, that is, $2l+1$ different orientations of L; thus, in the presence of a strong external magnetic field each energy level nl of the atom is split into $2l+1$ levels. This is called the normal *Zeeman effect*, which is explained by, and which in turn experimentally confirms, the quantization of angular momentum.

Well, if we adhere to the hypothesis, resulting from the abductive reasoning presented in steps one and two, that the electron has an intrinsic angular momentum, that is, a spin S, and, because it is a charged particle that rotates it also has an *intrinsic* magnetic moment M_S, then without stretching the imagination too much, we apply reasoning by analogy to assume both that S is quantized,

$$S = \sqrt{s(s+1)}\,\hbar$$

and $S_z = m_s\hbar$, analogously to how orbital angular momentum is handled in the formalism. Finally, as experimentally, Stern-Gerlach effect, the number of spatial orientations is $2s + 1 = 2$, then $s = 1/2$ and $m_s = \pm 1/2$.

Conclusion: The double orientation of the electron spin with respect to L produces the doubling of the energy levels of the hydrogenoid atoms and therefore the doubling of the spectral lines. This duplication is a consequence of the spin-orbit interaction existing between L and the spin magnetic dipole moment, as long as the acting magnetic field is not intense and it cannot cancel this interaction. The result is the observed *anomalous* Zeeman effect.

If we assimilate the reasoning by analogy implicit in this third step to abductive reasoning, since the assumed hypothesis facilitates an excellent quantum mechanical handling of the phenomenon, then we see that the hypothetical explanation of the anomalous Zeeman effect proceeds through a concatenation of successive abductions that complement each other step by step.

Another completely different issue is that of its *theoretical explanation*, which as we have already mentioned, takes place within the framework of Dirac's relativistic quantum mechanics. But even if we did not have it, the triple abduction presented here gives the phenomenon that triggers it a reassuring explanation.

Regardless of this historical and philosophical transcript, once the concept of spin is incorporated into the exclusion principle, its value for the theoretical explanation of the chemical valence of the elements is decisive. Indeed, Pauli's principle excludes that two fermions, in particular two electrons, can be in the same quantum state, i.e., have exactly the same four quantum numbers, since they have to be distinguished at least by their spin. For example, if two electrons fill a subshell *s* of the electronic configuration of an atom, their respective spins cannot have the same orientation: one will be oriented "up" and the other "down"; and if five electrons are found in a subshell or *p* orbital, four will be paired two by two and in each pairing, one will also be oriented "up" and the other towards "down," while the fifth electron will be unpaired. Now, what the Pauli exclusion principle requires is that in the fundamental or minimum energy state of an atom, the electrons in each orbital are paired; but, if not all of them are, those that are unpaired in the different orbitals that the energy level *n* allows, they will be the valence electrons of the element in question.

Not even Deductive Reasoning in Physics is Free from the Use of Imagination

Scientific creativity—normally linked to the use of conjectures or daring hypotheses that, with luck, end up seeming to respond to the observations or experimental results that trigger them—is not exhausted in the implementation of the various ways that take the form of reasoning generically known as *ampliative* reasoning: induction, abduction, reasoning by analogy, hypothetical trial, perhaps also serendipity, etc. Deductive reasoning, when used inter-theoretically, is also perfectly capable of serving the purposes of theoretical innovation. This peculiar form of reasoning is called *theoretical preduction* or simply *preduction*.

Indeed, although preductive reasoning is purely deductive, its specific peculiarity is that it is inter-theoretical, that is, for its implementation it is necessary to resort to accepted results—although not necessarily assumed to be true—proceeding from different theories and/or disciplines, which, taken as premises of the reasoning that is going to be initiated, allow—through their mathematical combination and guaranteeing the dimensional homogeneity of the procedure—to reach a result that, due to its novelty and/or explanatory capacity, may be interesting for scientific development.

In the choice of premises, luck can play a decisive role in the success of the process. In several works, I have illustrated the use of preductive reasoning in physics for innovation and explanation (Rivadulla, 2016, 2022). The example I present below shows the implementation of preduction for the purposes of scientific discovery or innovation.

Lee Smolin (2001, p. 166) claims that "there is a minimum value to the uncertainty in position, and this means that there is an absolute limit to the precision with which any object can be located in space" and Carlo Rovelli (2007, p. 1289) confirms that

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"gravity, relativity and quantum theory, taken together, appear to prevent position to be determined more precisely than the Planck scale." Obtaining this result, which comes from the combination of the Special Theory of Relativity, the General Theory of Relativity, and Quantum Mechanics, constitutes a typical case of preductive reasoning:

1. Assuming Heisenberg's Principle

$$\Delta x \Delta p \geq \frac{\hbar}{2}, \text{ i.e., } \Delta x \text{ x } m\Delta p \geq \frac{\hbar}{2}$$

and, in a simplified manner, that *x* and v are the indeterminacies in the position and velocity respectively of a particle, it turns out that

$$m\text{v}x \geq \frac{\hbar}{2} \text{ and } x \geq \frac{\hbar}{2m\text{v}}$$

2. In General Relativity formula

$$r_g = \frac{2Gm}{c^2}$$

has length dimensions and designates the gravitational radius of a body (Landau & Lifshitz, 1992, p. 398). Naming $r_g = x$ it results that

$$x = \frac{2Gm}{c^2}$$

3. From 1:

$$x \geq \frac{\hbar}{2m\text{v}} \text{ and 2: } x = \frac{2Gm}{c^2}\text{, it results that } x^2 \geq \frac{2Gm}{c^2}\frac{\hbar}{2m\text{v}}$$

4. Now bringing into play the Special Theory of Relativity, according to which all speed v is limited by the speed of light, so that $0 \leq \text{v} \leq \text{c}$, taking $\text{v} \approx c$, from 3 it turns out that

$$x^2 \geq \frac{G}{c^2} \cdot \frac{\hbar}{c}\text{, i.e., } x \geq \sqrt{\frac{G\hbar}{c^3}}$$

5. Now, in the Planck Unit System

$$\ell_p = \sqrt{\frac{G\hbar}{c^3}} = 10^{-35}$$

m is known as *Planck length*.

6. But from 4 and 5 it turns out that $x \geq \ell_p$. That is to say: the indeterminacy of the position of a particle moving at relativistic speed is limited by the Planck length. This indeterminacy can never be smaller than ℓ_p, which could be assumed as the minimum length in Nature, and suggest the attractive hypothesis of the discontinuity of space.

It is totally reasonable to start with the Heisenberg principle, since it itself incorporates the idea of minimal indeterminacy in the position of a particle of mass *m* given by

$$\Delta x \geq \frac{\hbar}{2m\text{v}}$$

Darío Maravall (1950, pp. 249–250) already deduced, combining this principle with the special theory of relativity, that is, by reasoning preductively, that we can speak of a quantization of space and time for any relativistic particle, in particular for an electron of mass $m_o = 9{,}1 \times 10^{-28}\, g$: "it makes no sense in wave mechanics to talk about lengths less than $h / m_o c^2$ or of time durations less than ." Specifically, $h / m_o c = 2{,}4 \times 10^{-10}\, cm$, which by the way is about 23 orders of magnitude larger than .

Anyway, the question was how to combine the quantum mechanical formula of the indeterminacy of the position with available accepted results, so that it might end up being related to Planck length. And this is where luck/imagination plays an important role. Well, if we could find the appropriate theoretical result that can be introduced into the reasoning, fulfilling with the requirement of dimensional homogeneity, it could lead us to the result that the minimum indeterminacy in the position of a particle, independent of its mass and any another magnitude, is given in terms of Planck length. Theoretical development effectively puts at our disposal the *Schwarzschild radius* formula, whose dimensions perfectly adjust to deducing the mentioned result: that the minimum indeterminacy in the position of any particle is the Planck length, and perhaps also the imaginative hypothesis of space discontinuity.

Conclusion

Theoretical physics is a human construction for the achievement of which the image that perhaps best suits it is that of a "convective" process that comes from experience, is consolidated as theoretical reasoning, descends back to experience, observation and experiment, and rises again to the theoretical level. It is like a wheel in constant motion. Sometimes, what is consolidated as theoretical is the result of experimental confirmation about which there is no longer any doubt. When this happens, the empirical basis, imaginative force, and luck, to which certain theoretical concepts owe their origin, is forgotten. It only counts that it is a theoretical element that has become indispensable from a given moment on. This is why for the methodology of science, the process that allows the postulation of a new idea that is, its discovery, is as important as its empirical or experimental confirmation.

In this paper I have intentionally highlighted expressions that make clear situations of conjectures, trials, hypotheses that are raised with explanatory intent. Thus, Planck, for example, begins by *assuming* a deep relationship of dependence of entropy on energy and he ends up being *attracted* by one of the different relationships *arbitrarily constructed* for this purpose. Actually, luck led him to opt for an interpolation formula. Its limited value would end up leading him, after changing in favour of the relationship between entropy and probability, to the establishment of the elementary quantum of action. But for Sommerfeld, the proposal of the hypothesis of energy quanta was still a *daring step*, which nevertheless allowed Einstein to offer a *simple explanation* of the observations and Bohr his atomic theory as well.

That the energy of light is distributed in space discontinuously through a finite number of energy quanta is presented by Einstein as an *assumption*. And Planck's elementary quantum of action provided Bohr with the basis for a satisfactory explanation of the atomic constitution. Although Bohr himself recognizes the *hypothetical* nature of his assumptions, Planck's theory offered Bohr the best explanation of experimental facts compared to classical electrodynamics. And while for Sommerfeld Bohr's atomic postulates may at first glance seem *grotesque*, they were also supported empirically.

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Finally, Pauli's almost *mysterious* postulation of the existence of four quantum numbers and the exclusion principle constitutes, despite its plausibility, a *daring hypothesis*, since at the time it lacked theoretical justification. But as they were quickly accepted due to their explanatory value, this postulation must be considered an abductive hypothesis or inference to the best explanation. This also applies to the existence of electron spin, whose theoretical justification by Dirac still had to wait a while.

In short, primitive quantum and atomic physics began their journey by dint of trials, conjectures and imaginative hypotheses, whose insertion into the theoretical corpus was done hand in hand with the tests that, experimental physicist by experimental physicist, showed their explanatory indispensability, consolidating them little by little in the theoretical context of the time. And, as we have seen in the last episode, inter-theoretical deductive reasoning is not alien to the use of imagination either.

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